π^*

The Two-Point Mixture Index of Model Fit
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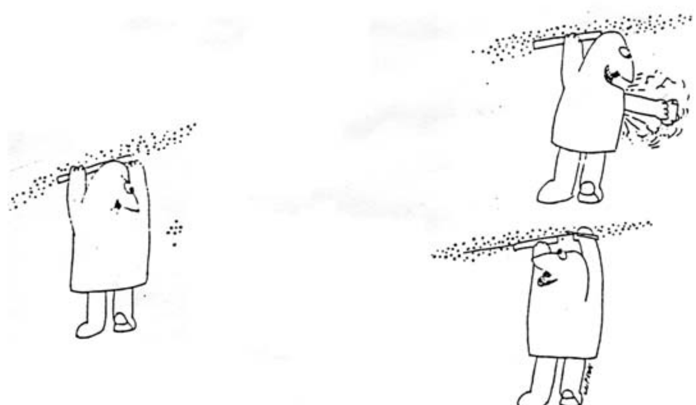

Correlation and Regression Analysis

Two-Point Mixture Index of Fit

Let P be the "true" distribution for the cell proportions in a frequency table. Rudas, Clogg and Lindsay (1994) {RCL hereafter} propose a two-point mixture model:

$$P = (1-\pi)\cdot\Phi + \pi\cdot\Psi \qquad [1]$$

where

Φ	=	probability distribution implied by probabilistic model, H
Ψ	=	an arbitrary, unspecified probability distribution
$0 \le \pi \le 1$	=	proportion of the population not consistent with H

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Comparison with Chi-Square Statistic

For the two-point mixture model, the "expected" proportions, associated with H are always equal to or less than corresponding observed proportions [$\hat{P}_{ij} \le P_{ij}$] and, of course, the same is true for expected and observed frequencies.

Note that this representation is different than the usual "fit" and "lack-of-fit" components associated with chi-square goodness-of-fit procedures. For example, for a two-way frequency table, let E_{ij} represent theoretical expected frequencies based on, say, an independence model. In general, E_{ij} may be less than, equal to or greater than the corresponding observed value, O_{ij} .

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Definition of the Fit Index

π in Equation [1] is not unique and the equation is true, *de facto*, for any model for any frequency table.

For independence hypothesis, delete 2 from cell A2B2

	B1	B2
A1	15	30
A2	10	22

	B1	B2
A1	1	1
A2	1	1

But this table omitting 73 cases also fits independence hypothesis

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Definition of the Fit Index – Cont'd

The index of fit, π^*, is defined as the **smallest** value of π for which the representation in Equation [1] holds:

$$\pi^* = \inf\{\pi \mid P = (1-\pi)\cdot\Phi + \pi\cdot\Psi, \Phi \in H\}$$

π^* can be interpreted as the minimum proportion of cases that must be omitted from the frequency table in order to provide perfect fit to the remaining data.

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Properties of the Fit Index

π^* is unique

π^* is defined on the 0,1 interval

For nested models, π^* has the property of decreasing (actually, never increasing) in magnitude for increasingly more complex models

π^* is invariant if frequencies are increased/decreased by an arbitrary multiplicative factor

Estimation for 2x2 Table

	B1	B2
A1	15	30
A2	10	22

	B1	B2
A1	a	b
A2	c	d

If $ad > bc$, then $\hat{\pi}^* = \dfrac{ad - bc}{an}$ for independence

and the expected cell frequencies for the model are $\{a, b, c, bc/a\}$

E. g., $\hat{\pi}^* = \dfrac{15 \times 22 - 30 \times 10}{15 \times 77} = \dfrac{30}{1155} = \dfrac{2}{77} = .026$

Estimating π^*

MLE: A method suggested by RCL entrails a guided search that, at each step, sets the value of π^* and derives maximum likelihood estimates (MLE's) of the parameters in the components of Equation (1) using an EM algorithm. The process can start with a small value (e.g., .005) and increment by a small constant (e.g., .005) with re-estimation at each step. At some step (and beyond), the value of the likelihood-ratio chi-square fit statistic, G^2, becomes (nearly) 0 and this is the final estimate of the fit index. See RCL for details.

NLP: Non-linear programming is a directed-search technique first recommended by Xi (1994) and Xi & Lindsay (1996). For relatively simple applications, the SOLVER procedure in Excel (with some tweaking) can be used as illustrated in this presentation (see Dayton, 2002, for details). NLP routines are available in SAS, Gauss, etc.

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NLP Computing Steps for Frequency Data

Provide start values for the parameters $\theta = \{\theta_m\}$ and include the sum of expected frequencies, $\sum_{j=1}^{J} n_j^* \le N$ as a parameter for the NLP algorithm.

Define expected frequencies for the cells of the frequency table using the model, H, based on θ.

Impose the restrictions $n_j^* \le n_j$ for all j; in addition, impose relevant restrictions on the parameters (e.g., non-negativity); in Microsoft Excel Solver, these are termed *Constraints*.

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NLP Computing Steps for Frequency Data – cont'd

Define the objective function to be maximized as the sum of the expected frequencies, $\sum_{j=1}^{J} n_j^*$; in Microsoft Excel Solver, this is called the *Target Cell*; at convergence of the NLP algorithm,

$$\hat{\pi}^* = 1 - \sum_{j=1}^{J} n_j^* / N$$

NOTE: These steps can be implemented using, for example, Excel Solver or Gauss sqpsolve routine. See Dayton (1999, 2002) for more details.

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Estimating a Lower Bound for the Two-Point Mixture Index

The estimate, $\hat{\pi}^*$, is subject to random fluctuation due to the peculiarities of sample data. In general, $\hat{\pi}^*$ may overestimate lack of fit. RCL derived a lower confidence bound, $\hat{\pi}_L$, based on a G^2 fit statistic equal to 2.70 (i.e., the 90th percentage point of the chi-square distribution with one degree of freedom). Their program, Mixit, can be used to find the lower limit by the same iterative procedure used to compute $\hat{\pi}^*$. The confidence interval is one-sided since all values of $\hat{\pi}$ greater than $\hat{\pi}^*$ yield models of the form of equation [1] that fit the observed frequencies perfectly (i.e., $G^2 = 0$ if $\hat{\pi} > \hat{\pi}^*$). For more general data situations than those that can be fit by Mixit, the standard error of $\hat{\pi}^*$ can be estimated using re-sampling techniques (e.g., the jackknife; see Dayton, 1999, Dayton 2002). Clogg, Rudas & Xi (1995) suggest that the difference, $\hat{\pi}^* - \hat{\pi}_L$, provides a measure of the effect of sample size on the estimator, $\hat{\pi}^*$.

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Guidelines (?)

There is no general guideline for interpreting the two-point mixture index but, intuitively, values of 10% to 5% or less seem small. RCL remark that 10% is “reasonable” for a specific 4x4 cross-classification table but there is no absolute standard for the index that represents acceptable fit in all settings. In particular, 10% for the first example in the RCL paper represents only about 59 respondents whereas it represents about 2526 respondents for their second example.

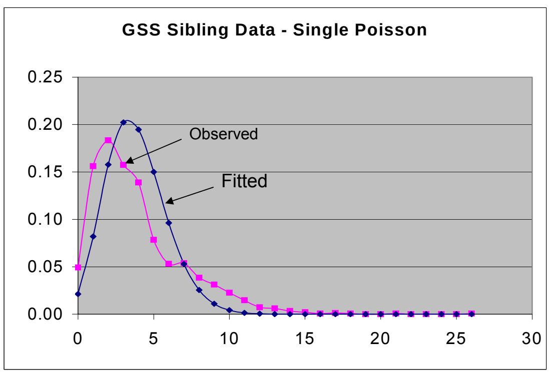
Note on 0 Observed Frequencies

Fr an independence model in a two-way contingency table, the existence of a 0 observed frequency in a cell requires that a corresponding row or column proportion be equation to 0. In effect, this creates 0 expected frequencies for the entire row or column. Among the approaches for avoiding this undesirable result are: (a) replace 0 cell frequencies with small, flattening values (e.g., .1 or .5); or, (b) treat the 0 as a structural 0 and explicitly model the structure.

First Example:

Number of Siblings
GSS self report

# Sibs	Count	Poisson1	Prop
0	74	0.0213	0.0492
1	235	0.0819	0.1561
2	276	0.1577	0.1834
3	237	0.2024	0.1575
4	209	0.1948	0.1389
5	118	0.1500	0.0784
6	80	0.0962	0.0532
7	81	0.0529	0.0538
8	58	0.0255	0.0385
9	47	0.0109	0.0312
10	34	0.0042	0.0226
11	22	0.0015	0.0146
12	11	0.0005	0.0073
13	9	0.0001	0.0060
14	5	0.0000	0.0033
15	3	0.0000	0.0020
16	1	0.0000	0.0007
17	2	0.0000	0.0013
18	1	0.0000	0.0007
19	0	0.0000	0.0000
20	0	0.0000	0.0000
21	1	0.0000	0.0007
22	0	0.0000	0.0000
23	0	0.0000	0.0000
24	0	0.0000	0.0000
25	0	0.0000	0.0000
26	1	0.0000	0.0007
	1505		1.0000



$$P(y) = \frac{\lambda^y \cdot e^{-\lambda}}{y!}$$

where λ is the rate parameter;
estimated as the mean of y (3.93)

		GSS Number of Siblings Data						
		Single Poisson Process						
	Observed		Single Poisson		Two-Point Mixture			
# Sibs	**Freq**	**Prop**	**E(Prop)**	**E(Freq)**	**E(Prop)**	**E(Freq)**		
0	74	0.049	0.020	29.56	0.069	74.00		
1	235	0.156	0.077	116.19	0.184	198.21		
2	276	0.183	0.152	228.30	0.246	265.45		
3	237	0.157	0.199	299.08	0.220	237.00		
4	209	0.139	0.195	293.84	0.147	158.70		
5	118	0.078	0.153	230.96	0.079	85.02		
6	80	0.053	0.101	151.28	0.035	37.95		
7	81	0.054	0.056	84.93	0.014	14.52		
8	58	0.039	0.028	41.72	0.005	4.86		
9	47	0.031	0.012	18.22	0.001	1.45		
10	34	0.023	0.005	7.16	0.000	0.39		
11+	56	0.037	0.002	3.74	0.000	0.12		
	1505	1.000	1.000	1505.00	1.000	1077.66		
			G^2 =	586.962	$\hat{\pi}^*$ =	0.284	$\hat{\pi}^*_L$ =	0.221
			λ =	3.93	λ =	2.68		

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Jackknife SE							
# Sibs	**Count**	**Prop**	**PoissP***	**E(Freq)**	**J(π*)**	**Wt(SS)**	
0	74	0.049	0.069	74.00	**0.2845**	2.22E-05	
1	235	0.156	0.184	198.21	**0.2835**	5.33E-05	
2	276	0.183	0.246	265.45	**0.2835**	6.26E-05	
3	237	0.157	0.220	237.00	**0.2862**	0.00117	
4	209	0.139	0.147	158.70	**0.2835**	4.74E-05	
5	118	0.078	0.079	85.02	**0.2835**	2.67E-05	
6	80	0.053	0.035	37.95	**0.2835**	1.81E-05	
7	81	0.054	0.013	14.52	**0.2835**	1.84E-05	
8	58	0.039	0.005	4.86	**0.2835**	1.31E-05	
9	47	0.031	0.001	1.45	**0.2835**	1.07E-05	
10	34	0.023	0.000	0.39	**0.2835**	7.71E-06	
11+	56	0.037	0.000	0.12	**0.2835**	1.27E-05	
	1505	1.000	1.000	1077.66		0.00146	= VAR(J)
				π* =	**0.2839**	**0.0382**	**= SE(J)**

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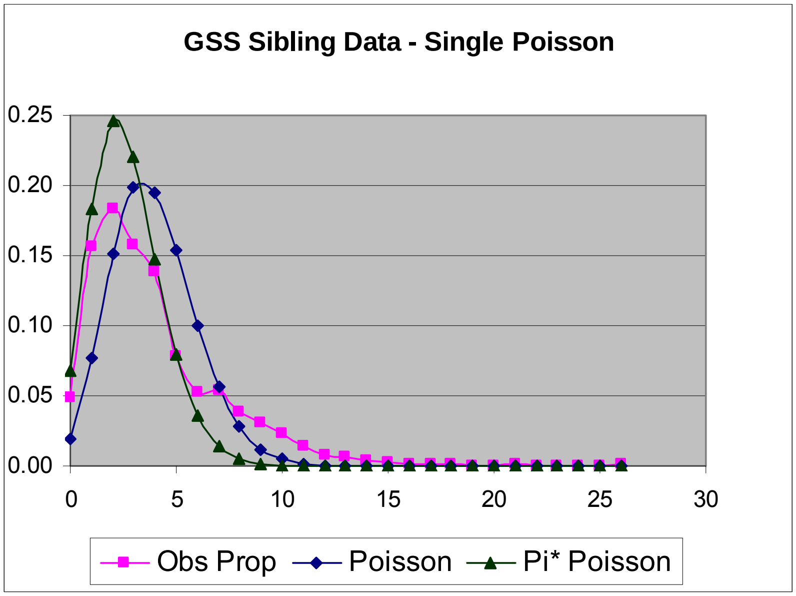

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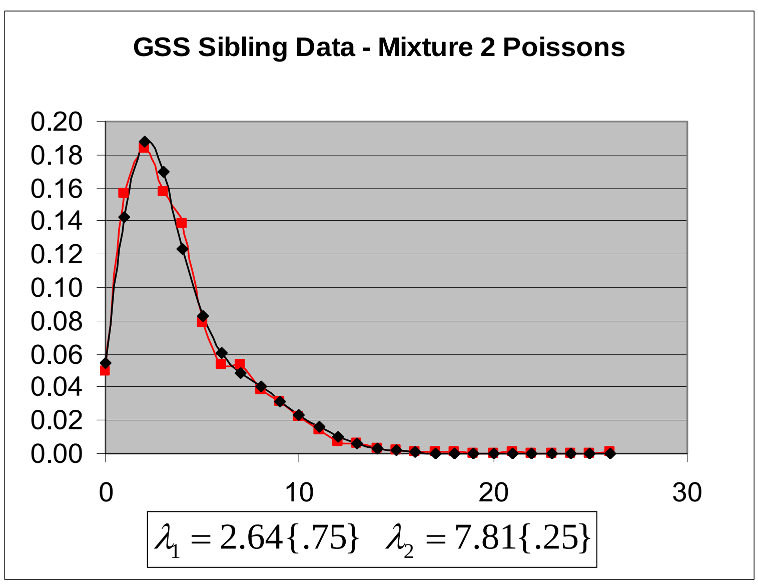

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		GSS Number of Siblings Data						
		Mixture of Two Poissons						
	Observed		**Mixture 2 Poissons**		**Two-Point Mixture**			
# Sibs	**Freq**	**Prop**	**E(Prop)**	**E(Freq)**	**E(Prop)**	**E(Freq)**		
0	74	0.049	0.054	80.70	0.054	74.00		
1	235	0.156	0.142	213.84	0.142	195.76		
2	276	0.183	0.190	285.35	0.189	260.94		
3	237	0.157	0.172	259.13	0.172	236.99		
4	209	0.139	0.124	186.69	0.124	171.21		
5	118	0.078	0.082	123.04	0.082	113.58		
6	80	0.053	0.057	85.98	0.058	80.00		
7	81	0.054	0.045	67.96	0.046	63.42		
8	58	0.039	0.038	57.11	0.038	53.11		
9	47	0.031	0.031	46.85	0.031	43.25		
10	34	0.023	0.024	35.88	0.024	32.84		
11+	56	0.037	0.042	62.46	0.041	56.00		
	1505	1.000	1.000	1505.00	1.000	1381.09		
			G^2 =	11.221	π^* =	0.081	π^*_L =	0.019
			λ =	2.64, 7.81	λ =	2.63, 7.74		
			θ =	.75, .25	θ =	.75, .25		

Second Example:

Maryland MSPAP Data

MSPAP π* for 3LCA Model

A	B	C	D	E	Freq	Prob	E(F)
0	0	0	0	0	1614	0.13	1614.00
1	0	0	0	0	594	0.05	594.00
0	1	0	0	0	375	0.03	375.00
1	1	0	0	0	262	0.02	262.00
0	0	1	0	0	89	0.01	89.00
1	0	1	0	0	109	0.01	93.36
0	1	1	0	0	50	0.00	47.50
1	1	1	0	0	99	0.01	84.22
0	0	0	1	0	1296	0.11	1296.00
1	0	0	1	0	1132	0.09	1132.00
0	1	0	1	0	568	0.05	568.00
1	1	0	1	0	810	0.07	810.00
0	0	1	1	0	335	0.02	222.13
1	0	1	1	0	662	0.04	448.83
0	1	1	1	0	285	0.02	285.00
1	1	1	1	0	936	0.08	935.67
0	0	0	0	1	108	0.01	108.00
1	0	0	0	1	86	0.01	86.00
0	1	0	0	1	53	0.00	43.87
1	1	0	0	1	82	0.00	59.41
0	0	1	0	1	22	0.00	16.50
1	0	1	0	1	52	0.00	32.21
0	1	1	0	1	29	0.00	20.28
1	1	1	0	1	61	0.01	60.60
0	0	0	1	1	328	0.02	189.31
1	0	0	1	1	387	0.02	297.66
0	1	0	1	1	274	0.01	175.78
1	1	0	1	1	566	0.04	500.99
0	0	1	1	1	131	0.01	114.01
1	0	1	1	1	389	0.03	389.00
0	1	1	1	1	277	0.02	276.44
1	1	1	1	1	1066	0.09	1066.00
					13127	1.00	12297.75

	Standard LCA			π* Solution		
	LC1	LC2	LC3	LC1*	LC2*	LC3*
A	0.836	0.209	0.582	0.798	0.067	0.506
B	0.801	0.163	0.390	0.742	0.114	0.313
C	0.741	0.010	0.314	0.709	0.010	0.129
D	0.957	0.300	0.805	0.946	0.214	0.661
E	0.594	0.035	0.273	0.549	0.028	0.129
θ	0.262	0.249	0.490	0.394	0.146	0.460
			N* =	12297.7		
			π* =	0.063		
	Note:					
	G^2 =	57.82				

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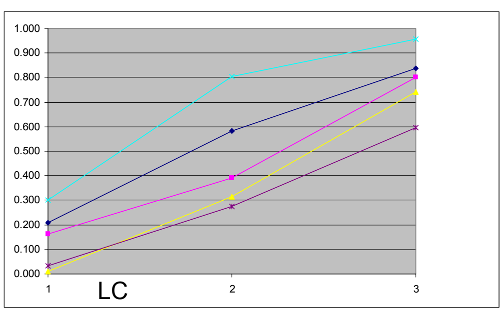
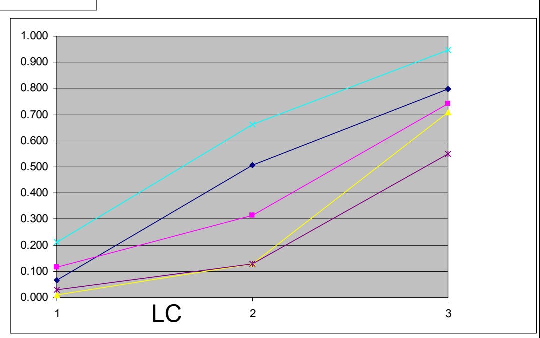


Standard LCA: Monotone but not doubly-monotone

π^* LCA



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MSPAP - Pi* for Rasch (1PL) Model
First 5 items from 17 item

1	2	3	4	5	Freq*	Prob*	E(F)	Pi* Solution θ	Pi* Solution δ	Standard Solution θ	Standard Solution δ
0	0	0	0	0	1614	0.123	1614.00	-2.17	1.55	-1.88	1.42
1	0	0	0	0	594	0.023	300.74	-1.06	0.71	-1.15	0.81
0	1	0	0	0	375	0.010	129.50	-0.33	0.34	-0.43	0.30
1	1	0	0	0	262	0.009	113.11	0.72	3.01	0.59	2.40
0	0	1	0	0	89	0.007	89.00	2.63	0.00	2.49	0.00
1	0	1	0	0	109	0.006	77.74	-0.38		-0.35	
0	1	1	0	0	50	0.003	33.47				
1	1	1	0	0	99	0.004	55.63	Pi* =	0.128		
0	0	0	1	0	1296	0.099	1296.00				
1	0	0	1	0	1132	0.086	1132.00				
0	1	0	1	0	568	0.037	487.43				
1	1	0	1	0	810	0.062	810.00				
0	0	1	1	0	335	0.026	335.00				
1	0	1	1	0	662	0.042	556.70				
0	1	1	1	0	285	0.018	239.71				
1	1	1	1	0	936	0.062	814.19				
0	0	0	0	1	108	0.005	61.87				
1	0	0	0	1	86	0.004	54.04				
0	1	0	0	1	53	0.002	23.27				
1	1	0	0	1	82	0.003	38.67				
0	0	1	0	1	22	0.001	15.99				
1	0	1	0	1	52	0.002	26.58				
0	1	1	0	1	29	0.001	11.44				
1	1	1	0	1	61	0.003	38.87				
0	0	0	1	1	328	0.018	232.88				
1	0	0	1	1	387	0.029	387.00				
0	1	0	1	1	274	0.013	166.64				
1	1	0	1	1	566	0.043	566.00				
0	0	1	1	1	131	0.009	114.53				
1	0	1	1	1	389	0.030	389.00				
0	1	1	1	1	277	0.013	167.50				
1	1	1	1	1	1066	0.081	1066.00				
					13127	0.872	11444.48				

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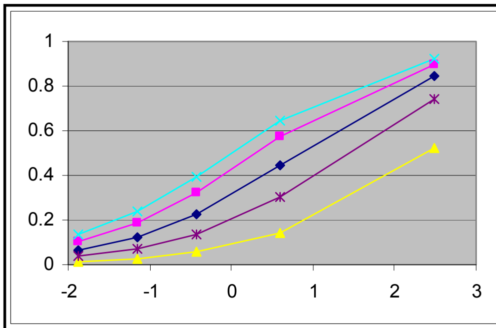
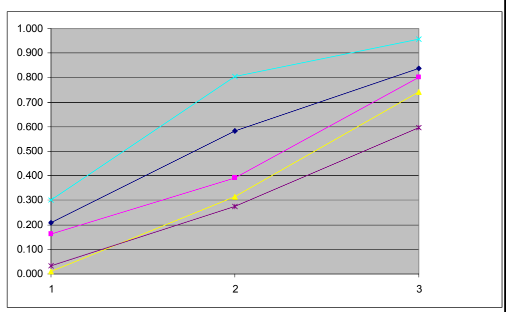


PL1 = Rasch

LC3



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Note: The Rasch model estimated by conditional MLE is equivalent to a restricted latent class model. In particular for M items, a restricted LC model with RE[M+.5)/2] classes provides the same fit as a Rasch model. The restrictions on the LC model require that the item conditional probabilities are ordered AND fall along parallel logistic functions. A program incorporating these restrictions, PRASCH, described in Lindsay, Clogg and Grego (1991).

Third Example:

Self Report - Academic Cheating

					Rasch Model fit to Cheating Items							
									Parameter Estimates			
					Standard		Two-Point					
Cheating Item					Rasch Model		Mixture		Standard Rasch Estimates			
A	B	C	D	Freq	E(Freq)	G^2	E(Freq)		Θ	δ		PML δ
0	0	0	0	207	207.00	0.00	207.00	1	-4.48	-0.92		0.13
1	0	0	0	10	13.82	-3.24	10.00	2	-1.23	-0.77		-0.06
0	1	0	0	13	16.11	-2.79	13.00	3	-0.91	-1.54		0.82
1	1	0	0	11	2.95	14.48	0.85	4	-0.38	0.11		-0.90
0	0	1	0	7	7.40	-0.39	7.00	5	0.04		r =	-0.999
1	0	1	0	1	1.35	-0.30	0.46					
0	1	1	0	1	1.58	-0.46	0.59		Two-Point Mixture Estimates			
1	1	1	0	1	0.79	0.24	0.43		Θ	δ		
0	0	0	1	46	38.67	7.99	46.00	1	0.44	-3.17		
1	0	0	1	3	7.08	-2.58	3.00	2	-0.02	-2.90		
0	1	0	1	4	8.25	-2.90	3.90	3	0.25	-3.52		
1	1	0	1	4	4.12	-0.12	2.86	4	1.32	-1.64		
0	0	1	1	5	3.79	1.39	2.10	5	1.94		r =	0.975
1	0	1	1	2	1.89	0.11	1.54	N*	302.73			
0	1	1	1	2	2.20	-0.19	2.00					
1	1	1	1	2	2.00	0.00	2.00					
		Total		319	319.00	22.50	302.73	π^* =	0.051			

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Fourth Example:

Continuous Variables

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L_∞ regression and mixture index of fit

A. Low
ELTE Institute of Sociology, Hungary

"In a general case an n dimension regression subspace can only fit to n points, that is it can describe n points perfectly. In this case the view of mixture index of fit cannot be applied because an n dimension subspace can be fit to the subset with any n element of the observed points. This kind of decomposition cannot be divided into two parts arbitrary only into one part which has exactly n element in it and to an other part with the rest. All the above mentioned solutions does not lead to the description of the observed distribution.

For the decomposition of the observed distribution into two parts —one of them is based on the model and the other one is unrestricted— we have to forget the perfect fit of the model and therefore a new error term e must be again introduced."

Unpublished manuscript found at:

www.math.uni-klu.ac.at/stat/Tagungen/Ossiach/Low.pdf

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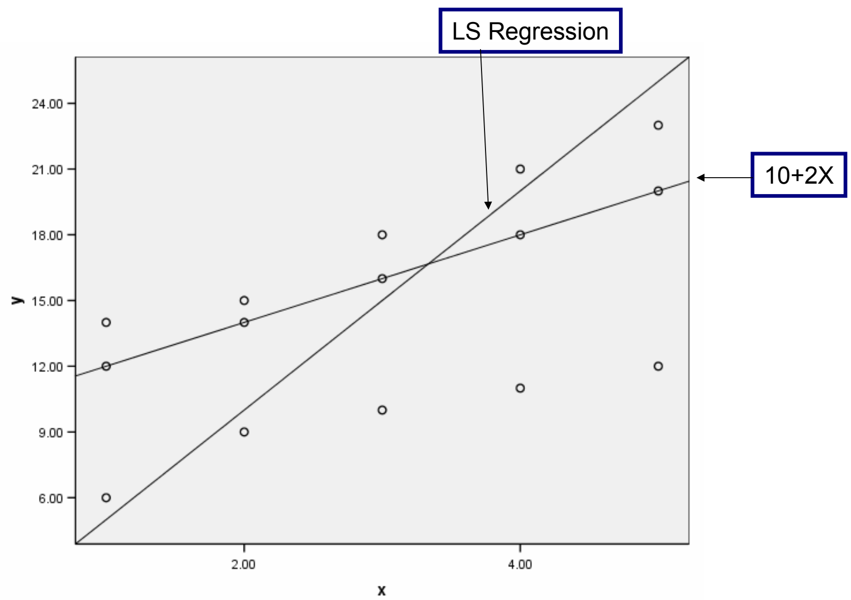

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State Level Data from JSE

Correlations

		SAT V	SAT Q
SAT V	Pearson Correlation	1	.970**
	Sig. (2-tailed)	.	.000
	N	50	50
SAT Q	Pearson Correlation	.970**	1
	Sig. (2-tailed)	.000	.
	N	50	50

**. Correlation is significant at the 0.01 level

← Original Scores

satv2 * satq2 Crosstabulation

Count

		satq2		Total
		1.00	2.00	
satv2	1.00	23	0	23
	2.00	1	26	27
Total		24	26	50

$$H:\rho=1 \qquad \hat{\pi}^{*}=1/50=.02$$

Correlations

		satv2	satq2
satv2	Pearson Correlation	1	.961**
	Sig. (2-tailed)	.	.000
	N	50	50
satq2	Pearson Correlation	.961**	1
	Sig. (2-tailed)	.000	.
	N	50	50

**. Correlation is significant at the 0.01 level

↑ Dichotomized Scores

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Correlations

		SAT V	SAT Q	P/T ratio	T SALARY
SAT V	Pearson Correlation	1	.970**	.064	-.477**
	Sig. (2-tailed)	.	.000	.660	.000
	N	50	50	50	50
SAT Q	Pearson Correlation	.970**	1	.095	-.401**
	Sig. (2-tailed)	.000	.	.510	.004
	N	50	50	50	50
P/T ratio	Pearson Correlation	.064	.095	1	-.001
	Sig. (2-tailed)	.660	.510	.	.994
	N	50	50	50	50
T SALARY	Pearson Correlation	-.477**	-.401**	-.001	1
	Sig. (2-tailed)	.000	.004	.994	.
	N	50	50	50	50

**. Correlation is significant at the 0.01 level (2-tailed).

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Correlations

		satv2	satq2	ptratio2	tsalary2
satv2	Pearson Correlation	1	.961**	.157	-.441**
	Sig. (2-tailed)	.	.000	.275	.001
	N	50	50	50	50
satq2	Pearson Correlation	.961**	1	.199	-.400**
	Sig. (2-tailed)	.000	.	.167	.004
	N	50	50	50	50
ptratio2	Pearson Correlation	.157	.199	1	.160
	Sig. (2-tailed)	.275	.167	.	.267
	N	50	50	50	50
tsalary2	Pearson Correlation	-.441**	-.400**	.160	1
	Sig. (2-tailed)	.001	.004	.267	.
	N	50	50	50	50

**. Correlation is significant at the 0.01 level (2-tailed).

satv2 * tsalary2 Crosstabulation

Count

		tsalary2		Total
		1.00	2.00	
satv2	1.00	6	17	23
	2.00	19	8	27
Total		25	25	50

$H:\rho=-1 \qquad \hat{\pi}^*=14/50=.28$

satv2 * ptratio2 Crosstabulation

Count

		ptratio2		Total
		1.00	2.00	
satv2	1.00	13	10	23
	2.00	11	16	27
Total		24	26	50

$$H:\rho=o \qquad \hat{\pi}^*=(13\times15-10\times11)/(13\times50)$$
$$=85/650=.13$$

Summary

- π^* is a widely applicable model-fit index for models based on frequency data.
- π^* has a simple and intuitive interpretation in terms of deleted observations.
- π^* is easy to compute only for very simple cases such as 2x2 contingency tables.
- π^* may be generalized to models based on continuous data but this requires an arbitrary recoding of data into categories.
- Unresolved issues with π^* include (a) efficient computational approaches; (b) effectiveness of different approaches for dealing with 0 observed frequencies; (c) efficient methods for assessing sampling variability.

References

Clogg, C. C., Rudas, T., Xi, L. (1995) A new index of structure for the analysis of models for mobility tables and other cross-classifications. In P. Marsden (ed.) *Sociological Methodology 1995*, 197-222, Blackwell, Oxford.

Dayton, C. M. (1999). *Latent Class Scaling Analysis*. Sage Publications.

Dayton, C. M. (2002). Applications and Computational Strategies for the Two-Point Mixture Index of Fit. Br*itish Journal of Mathematical & Statistical Psychology*, In Press.

Dayton, C. M. & Macready, G. B. (1980). A scaling model with response errors and intrinsically unscalable respondents, *Psychometrika, 45*, 343 - 356.

Goodman, L. A. (1975). A new model for scaling response patterns: An application of the quasi-independence concept. *Journal of the American Statistical Association, 70*, 755-768.

Lindsay, B., Clogg, C. C., and Grego, J. (1991). Semiparametric estimation in the Rasch model and related models, including a simple latent class model for item analysis. *Journal of the American Statistical Association*, 86, 96-107.

Rudas, T. (1999). The mixture index of fit and minimax regression. *Metrika*, 50, 163-172.

Rudas, T. (2002). A latent class approach to measuring the fit of a statistical model. In Hagenaars, J.A. & McCutcheon, A.L. *Applied Latent Class Analysis*, Cambridge University Press.

Rudas, T., Clogg, C. C. & Lindsay, B. G. (1994). A new index of fit based on mixture methods for the analysis of contingency tables. *Journal of the Royal Statistical Society, Series B, 56*, 623-639.

Rudas, T. & Zwick, R. (1997). Estimating the importance of differential item functioning. *Journal of Educational and Behavioral Statistics, 22*, 31-45.

References - Cont'd

Verdes, E., Rudas, T. (2002) The π* Index as a New Alternative for Assessing Goodness od Fit of Logistic Regression. In: Haitovsky, Y., Lerche, H. R., Ritov, Y. (eds.) Foundations of Statistical Inference. Springer, 167-177.

Xi, L. (1994). The mixture index of fit for the independence model in contingency tables. Master of Arts paper, Department of Statistics, Pennsylvania State University.

Xi, L. (1996). The mixture index of fit. PhD. Thesis, Department of Statistics, Pennsylvania State University.

Xi, L. & Lindsay, B. G. (1997). A note on calculating the index of fit for the analysis of contingency tables. *Sociological Methods & Research*, 25, 248-259.

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